Coordination of supply chain with revenue sharing contract in a fuzzy environment: Investigation and analysis

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Abstract

The coordination of a decentralized supply chain can be achieved by adopting supply chain contracts. This paper investigates how a decentralized supply chain can be coordinated by a revenue sharing contract, in which the supplier offers the retailer a lower wholesale price in return for a percentage of the retailer's revenue. Salvage value and goodwill loss are taken into consideration, since it is approximately close to the real world. Fuzzy variable is used to denote the uncertain demand of the market, and investigation is given to illustrate the relationship between the revenue sharing contract parameters in the fuzzy framework. What's more, analysis of the optimal revenue sharing contract is presented to explain the reason why the contract can maximize the total profit of the whole supply chain as well as uphold the supplier and the retailer's profit.

Keywords: coordination; decentralized supply chain; revenue sharing contract; uncertain demand; fuzzy variable

1 Introduction

Coordination supply chain has been a major issue in supply chain management research recently. If all players of the supply chain unite under the coordination of the coordination system, they can maximize both their profits and social benefits. However, it is difficult to put this ideal action into practice in the decentralized supply chain since it involves the existence of several decision makers pursuing different objectives, possibly conflicting among each other. Therefore, coordination of such supply chains must be normalized by feasible mechanisms. Such mechanisms include the supply chain contracts, through which the decision makers can unite their decisions to realize the performance improvement of the supply chain system and achieve whole-system benefit. Thus, it is becoming more and more significant to investigate supply chain contracts.

A supply chain contract is a mechanism which formally rule the transactions between the supply chain players. By adjusting the contract parameters, we can encourage the supply chain actors (suppliers, retailers, and so on) to unite under coordination and share the risk and revenue among themselves. Therefore, adopting supply chain contracts can achieve two main objectives: (1) to increase the total profit of the decentralized supply chain as close as to the profit of the centralized one; (2) to assure that every supply chain actors obtains a profit higher than he/she would gain without the contract.

Different types of supply chain contracts have been developed in the supply chain management literature. Generally, we can divide these contracts mainly into two kinds, namely, ordering-quantity contracts and pricing contracts. Among the ordering-quantity contracts, there are quantity flexibility contracts (Tsay, 1999), backup agreements (Eppen and Iyer, 1997) and so on. And quantity discounts (Weng, 1995), buy back or return polices (Emmons and Gilbert, 1998), revenue sharing contracts (Cachon and Lariviere, 2000) are the typical types of the pricing contracts.

In this paper, we mainly consider a specific type of supply chain contract, namely, the revenue sharing contract, which is adopted to coordinate a decentralized supply chain which faces an uncertain demand. Revenue sharing contracts have been adopted by many retail supply chains and have been proven to be effective in improving supply chain performance. Cachon and Lariviere

(2000) proposed the revenue sharing contract and studied its impact on supply chain performance. Mortimer (2000) estimated that the adoption of revenue sharing contracts increased the video-rental industry's total profit by seven percent.

A revenue sharing contract presented by Cachon and Lariviere can be described by the two parameters (ω, α); the supplier charges the retailer a unit wholesale price ω, lower than the unit cost c, in exchange for a percentage $(1-\alpha)$ of the retailer's revenue earned on the sold products. The retailer keeps a portion α of his revenue. By this revenue sharing contract, the decentralized supply chain can be coordinated, where the condition $\omega < c$ guarantees channel coordination whereas α determines the allocation of the total profit between the supplier and the retailer.

In this paper, we deal with a single-period decentralized supply chain involving one supplier and one retailer. In reality, the retailer may not have enough cash to make a one-time purchase to buy all the products he needs from the supplier, and a retailer will possibly lose many sales opportunities and his loss will result in loss opportunities for the supplier as well. If there are unsold products at the end of the selling period, the retailer will sell the leftover products at the salvage price; on the other hand, if the order can't meet the market demand, the retailer will suffer from goodwill loss, although the above situation is quite common in supply chain, but Cachon and Lariviere didn't take the salvage value and goodwill loss into account, and interestingly, by involving these two parameters into our model, we give the relationship between the contract parameters, namely, ω and α in this new setting. The results can be applied to applied to real situations. What's more, although by analyzing the adoption of a revenue sharing contract in a supply chain, Cachon and Lariviere pointed out that if a revenue sharing contract is in place, both players will be benefited while the overall profit of the whole system will be maximized, they didn't give deep analysis of the reason why the revenue sharing contract presented in their way could achieve the above goal. Thus, in our study, we not only give the relationship between the parameters, but also give the reason why such a revenue sharing contract can reach the aim.

Besides, we suppose that the supplier sells a new kind of product with a short life cycle, such as a new video, or a fashionable clothing to the retailer. Normally the market demand of this new product is uncertain and can't be precisely estimated by the retailer before he announces his order quantity to the supplier. Demand uncertainty may have deleterious effects on the players' decision, so treating the demand uncertainty appropriately is critical. In many supply chain management literatures, the uncertain demand is often assumed to be a random variable with known probability density function and cumulative distribution function. However, due to the lack of statistical observations of the demand parameters, some researchers realized that it is actually not reasonable for us to denote the uncertain demand by random variable, instead, they adopted fuzzy sets theory (Zadeh, 1965) to solve the supply chain problems in the fuzzy sense, and characterized the uncertain demand as fuzzy numbers. For instance, Yao, Chang and Ouyang (2000) considered a fuzzy inventory problem and denoted the total demand as a triangular fuzzy number. Recently, Liu (2002) laid a new foundation for optimization theory in uncertain environments, and characterized the imprecise parameters as fuzzy variables. In this paper, we use fuzzy variable to denote the uncertain market demand, and propose a fuzzy supply chain model under a revenue sharing contract. Besides, in order to calculate the expected value of profit functions of the supplier and the retailer, we give some theorems to calculate the expected value of the function of the fuzzy variable. Since these theorems haven't been proposed by others and is quite important for our study, it is necessary for us to present them prior to model formulation.

The main contributions of this paper are as follows; first, to formulate a fuzzy supply chain model with a revenue sharing contract considering the salvage value and goodwill loss to ensure supply chain coordination under decentralized control; second, to analyze the relationship between the parameters and give the reason why such a contract can achieve the goal of maximizing the overall profit of the whole supply chain while ensuring that each player can get his optimal profit.

The rest of the paper is organized as follows. In Section 2, we recall some basic concepts about fuzzy variables and give some important theorems on how to calculate the expected values of the fuzzy variables. Then we introduce the notations and assumptions required by the revenue sharing contract and formulate the fuzzy revenue sharing model in Section 3, we also analyze the proposed supply chain model and find the relationship between the contract parameters. Ultimately, in the last section, we make our conclusions about this paper.

2 Preliminaries on fuzzy variable

Let Θ be a nonempty set, $\mathcal{P}(\Theta)$ the power set of Θ, and Pos a possibility measure. Then the triplet $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ is called a possibility space.

Definition 1 (Nahmias 1978) *A fuzzy variable is defined as a function from a possibility space* $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ *to the set of real numbers.*

Based on the possibility measure Pos, the *credibility* (Cr) of a fuzzy event $\{\xi \geq r\}$ can be expressed by

$$\text{Cr}\{\xi \geq x\} = \frac{1}{2}(\text{Pos}\{\xi \geq x\} + 1 - \text{Pos}\{\xi < x\}), \tag{1}$$

then the membership function of ξ is derived from the credibility measure by

$$\mu(x) = (2\text{Cr}\{\xi = x\}) \wedge 1, \quad x \in \Re. \tag{2}$$

Definition 2 *(Liu 2002) Let ξ be a fuzzy variable defined on a possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$. Then the credibility distribution $\Phi : \Re \to [0, 1]$ of the fuzzy variable ξ is defined by*

$$\Phi(x) = \text{Cr}\{\theta \in \Theta \mid \xi(\theta) \leq x\}. \tag{3}$$

That is, $\Phi(x)$ is the credibility that the fuzzy variable ξ takes a value less than or equal to x.

Remark 1 *Let ξ be a fuzzy variable defined on a possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ with membership function μ. Then its credibility distribution*

$$\Phi(x) = \frac{1}{2}\left(\sup_{y \leq x} \mu(y) + 1 - \sup_{y > x} \mu(y)\right), \quad \forall x \in \Re. \tag{4}$$

Definition 3 *(Liu 2002) Let ξ be a fuzzy variable defined on a possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$. Then the credibility density function $\phi : \Re \to [0, +\infty)$ of the fuzzy variable ξ is a function such that*

$$\Phi(x) = \int_{-\infty}^{x} \phi(y)\text{d}y, \qquad \forall x \in \Re, \tag{5}$$

$$\int_{-\infty}^{+\infty} \phi(y)\text{d}y = 1, \tag{6}$$

where Φ is the credibility distribution of the fuzzy variable ξ.

Definition 4 *(Liu and Liu 2002) Let ξ be a fuzzy variable defined on the possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$. Then the expected value of ξ is defined as*

$$E[\xi] = \int_{0}^{+\infty} \text{Cr}\{\xi \geq r\}\text{d}r - \int_{-\infty}^{0} \text{Cr}\{\xi \leq r\}\text{d}r, \tag{7}$$

provided that at least one of the two integrals is finite.

Remark 2 *Let f be a function on $\Re \to \Re$ and ξ a fuzzy variable defined on the possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$, then the expected value of $f(\xi)$ is defined as*

$$E[f(\xi)] = \int_{0}^{+\infty} \text{Cr}\{f(\xi) \geq r\}\text{d}r - \int_{-\infty}^{0} \text{Cr}\{f(\xi) \leq r\}\text{d}r. \tag{8}$$

Lemma 1 *(Liu 2002) Let ξ be a fuzzy variable defined on the possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$, and $\phi(x)$ the credibility density function of ξ. If the Lebesgue integral $\int_{-\infty}^{+\infty} x\phi(x)\text{d}x$ is finite, then the expected value of ξ is*

$$E[\xi] = \int_{-\infty}^{+\infty} x\phi(x)\text{d}x. \tag{9}$$

Lemma 2 *(Liu 2002) Let ξ be a positive fuzzy variable with the support $[0,+\infty)$, and $\phi(x)$, $\Phi(x)$ the credibility density function and credibility distribution of ξ, respectively. Thereby the expected value of ξ is*

$$\begin{aligned} E[\xi] &= \int_0^{+\infty} x\mathrm{d}\Phi(x) \\ &= \int_0^{+\infty} x\phi(x)\mathrm{d}x. \end{aligned} \tag{10}$$

Considering more complex situations, we have the following theorems.

Theorem 1 *Let ξ be a positive fuzzy variable with a continuous membership function, and $f\colon \Re \to \Re$ a strictly increasing function with $\lim_{x\to+\infty} f(x) = M$ and $\lim_{x\to-\infty} f(x) = m$ $(m < M)$. If the Lebesgue integrals*

$$\int_0^{+\infty} \mathrm{Cr}\{f(\xi) \geq r\}\mathrm{d}r \quad \textit{and} \quad \int_{-\infty}^{0} \mathrm{Cr}\{f(\xi) \leq r\}\mathrm{d}r$$

are finite, then we obtain

$$E\left[f(\xi)\right] = \int_{f^{-1}(m)}^{f^{-1}(M)} f(r)\phi(r)\mathrm{d}r. \tag{11}$$

Proof. Since $f(x)$ is a strictly increasing function with $\lim_{x\to+\infty} f(x) = M$ and $\lim_{x\to-\infty} f(x) = m$, we can obtain $f^{-1}(x)$, the inverse function of $f(x)$, and we also have

$$\mathrm{Cr}\{f(\xi) \geq x\} \equiv 0,\ \mathrm{Cr}\{f(\xi) \leq x\} \equiv 1,\ \forall x \geq M \tag{12}$$

and

$$\mathrm{Cr}\{f(\xi) \leq x\} \equiv 0,\ \mathrm{Cr}\{f(\xi) \geq x\} \equiv 1,\ \forall x \leq M. \tag{13}$$

Thus

$$\begin{aligned} \int_0^{+\infty} \mathrm{Cr}\{f(\xi) \geq r\}\mathrm{d}r &= \int_0^{M} \mathrm{Cr}\{f(\xi) \geq r\}\mathrm{d}r \\ &= \int_0^{M} \mathrm{Cr}\{\xi \geq f^{-1}(r)\}\mathrm{d}r. \end{aligned}$$

Let $\tau = f^{-1}(r)$, then we have $r = f(\tau)$ and

$$\begin{aligned} &\int_0^{+\infty} \mathrm{Cr}\{f(\xi) \geq r\}\mathrm{d}r \\ &= \int_{f^{-1}(0)}^{f^{-1}(M)} \mathrm{Cr}\{\xi \geq \tau\}\mathrm{d}f(\tau) \\ &= \mathrm{Cr}\{\xi \geq \tau\}f(\tau)\big|_{f^{-1}(0)}^{f^{-1}(M)} - \int_{f^{-1}(0)}^{f^{-1}(M)} f(\tau)\mathrm{dCr}\{\xi \geq \tau\} \\ &= \mathrm{Cr}\{\xi \geq \tau\}f(\tau)\big|_{f^{-1}(M)} - \mathrm{Cr}\{\xi \geq f^{-1}(0)\}f\left(f^{-1}(0)\right) - \int_{f^{-1}(0)}^{f^{-1}(M)} f(\tau)\mathrm{dCr}\{\xi \geq \tau\} \\ &= \lim_{\tau\to f^{-1}(M)} \mathrm{Cr}\{\xi \geq \tau\}f(\tau) - \int_{f^{-1}(0)}^{f^{-1}(M)} f(\tau)\mathrm{dCr}\{\xi \geq \tau\}. \end{aligned} \tag{14}$$

Since $\mathrm{Cr}\{f(\xi) \geq M\} = 0$, we immediately get

$$\lim_{\tau\to f^{-1}(M)} \mathrm{Cr}\{\xi \geq \tau\}f(\tau) = \lim_{\tau\to f^{-1}(M)} \mathrm{Cr}\{f(\xi) \geq f(\tau)\}f(\tau) = \lim_{r\to M} \mathrm{Cr}\{f(\xi) \geq r\}r = 0, \tag{15}$$

where $r = f(\tau)$.

Then follows from (14) and (15), we obtain that

$$\begin{aligned}\int_0^{+\infty}\mathrm{Cr}\{f(\xi)\ge r\}\mathrm{d}r &= -\int_{f^{-1}(0)}^{f^{-1}(M)} f(\tau)\mathrm{d}\left(1-\mathrm{Cr}\{\xi\le\tau\}\right)\\ &= \int_{f^{-1}(0)}^{f^{-1}(M)} f(\tau)\mathrm{d}\mathrm{Cr}\{\xi\le\tau\}.\end{aligned}\tag{16}$$

Similarly, considering that the Lebesgue integral $\int_{-\infty}^{0}\mathrm{Cr}\{f(\xi)\le r\}\mathrm{d}r$ is finite, we have

$$\int_{-\infty}^{0}\mathrm{Cr}\{f(\xi)\le r\}\mathrm{d}r = -\int_{f^{-1}(m)}^{f^{-1}(0)} f(\tau)\mathrm{d}\mathrm{Cr}\{\xi\le\tau\}.\tag{17}$$

By (8), (16) and (17), the expected value of the function of fuzzy variable can be written as

$$\begin{aligned}E\left[f(\xi)\right] &= \int_{f^{-1}(0)}^{f^{-1}(M)} f(r)\mathrm{d}\mathrm{Cr}\{\xi\le r\} + \int_{f^{-1}(m)}^{f^{-1}(0)} f(\tau)\mathrm{d}\mathrm{Cr}\{\xi\le\tau\}\\ &= \int_{f^{-1}(m)}^{f^{-1}(M)} f(r)\mathrm{d}\mathrm{Cr}\{\xi\le r\}\\ &= \int_{f^{-1}(m)}^{f^{-1}(M)} f(r)\phi(r)\mathrm{d}r.\end{aligned}$$

The theorem is proved.

Theorem 2 *Let ξ be a fuzzy variable with a continuous membership function, and $f\colon \Re\to\Re$ a strictly increasing function. If the Lebesgue integrals*

$$\int_0^{+\infty}\mathrm{Cr}\{f(\xi)\ge r\}\mathrm{d}r \quad \textit{and} \quad \int_{-\infty}^{0}\mathrm{Cr}\{f(\xi)\le r\}\mathrm{d}r$$

are finite, then we have

$$E\left[f(\xi)\right] = \int_{-\infty}^{+\infty} f(r)\phi(r)\mathrm{d}r.\tag{18}$$

Proof. Since $f(x)$ is strictly increasing with $\lim_{x\to+\infty} f(x) = M$ and $\lim_{x\to-\infty} f(x) = m$, it is obvious that

$$\mathrm{Cr}\{\xi\le r\} = \mathrm{Cr}\{f(\xi)\le f(r)\},\tag{19}$$

and by Theorem 1, we know that

$$E\left[f(\xi)\right] = \int_{f^{-1}(m)}^{f^{-1}(M)} f(r)\mathrm{d}\mathrm{Cr}\{\xi\le r\}.\tag{20}$$

If $r\ge f^{-1}(M)$, it is easy to know that $f(r)\ge M$, then by (12),

$$\mathrm{Cr}\{\xi\le r\} = \mathrm{Cr}\{f(\xi)\le f(r)\} = 1,\ \forall r\ge f^{-1}(M).$$

If $r\le f^{-1}(m)$, it is easy to know that $f(r)\le m$, then by (13),

$$\mathrm{Cr}\{\xi\le r\} = \mathrm{Cr}\{f(\xi)\le f(r)\} = 0,\ \forall r\le f^{-1}(m).$$

Hence,

$$\int_{-\infty}^{f^{-1}(m)} f(r)\mathrm{d}\mathrm{Cr}\{\xi\le r\} = 0 \quad \text{and} \quad \int_{f^{-1}(M)}^{+\infty} f(r)\mathrm{d}\mathrm{Cr}\{\xi\le r\} = 0.\tag{21}$$

Then we have

$$
\begin{aligned}
E[f(\xi)] &= \int_{f^{-1}(m)}^{f^{-1}(M)} f(r)\mathrm{dCr}\{\xi \le r\} \\
&= \int_{f^{-1}(m)}^{f^{-1}(M)} f(r)\mathrm{dCr}\{\xi \le r\} + \int_{-\infty}^{f^{-1}(m)} f(r)\mathrm{dCr}\{\xi \le r\} + \int_{f^{-1}(M)}^{+\infty} f(r)\mathrm{dCr}\{\xi \le r\} \\
&= \int_{-\infty}^{+\infty} f(r)\mathrm{dCr}\{\xi \le r\} \\
&= \int_{-\infty}^{+\infty} f(r)\phi(r)\mathrm{d}r.
\end{aligned} \tag{22}
$$

The theorem is proved.
Furthermore, we have the following corollaries.

Corollary 1 *Let ξ be a fuzzy variable with a continuous membership function, and $f\colon \Re \to \Re$ a strictly monotonic function. If the Lebesgue integrals*

$$\int_0^{+\infty} \mathrm{Cr}\{f(\xi) \ge r\}\mathrm{d}r \quad \textit{and} \quad \int_{-\infty}^0 \mathrm{Cr}\{f(\xi) \le r\}\mathrm{d}r$$

are finite, then

$$E\left[f(\xi)\right] = \int_{-\infty}^{+\infty} f(r)\phi(r)\mathrm{d}r. \tag{23}$$

Corollary 2 *Let ξ be a positive fuzzy variable whose support is $[a, +\infty), (a > 0)$, and $f\colon \Re \to \Re$ a strictly monotonic function. If the Lebesgue integrals*

$$\int_0^{+\infty} \mathrm{Cr}\{f(\xi) \ge r\}\mathrm{d}r \quad \textit{and} \quad \int_{-\infty}^0 \mathrm{Cr}\{f(\xi) \le r\}\mathrm{d}r$$

are finite, then we have

$$E\left[f(\xi)\right] = \int_a^{+\infty} f(r)\phi(r)\mathrm{d}r. \tag{24}$$

Corollary 3 *Let ξ be a positive fuzzy variable whose support is [0,b], and $f\colon \Re \to \Re$ a strictly monotonic function. If the Lebesgue integrals*

$$\int_0^{+\infty} \mathrm{Cr}\{f(\xi) \ge r\}\mathrm{d}r \quad \textit{and} \quad \int_{-\infty}^0 \mathrm{Cr}\{f(\xi) \le r\}\mathrm{d}r$$

are finite, then we have

$$E\left[f(\xi)\right] = \int_0^{b} f(r)\phi(r)\mathrm{d}r. \tag{25}$$

3 The supply chain coordination under revenue sharing contract in a fuzzy setting

We consider a two-stage decentralized supply chain with one supplier and one retailer in a single period setting. Both the supplier and the retailer are assumed to be risk-neutral. The supplier is considered as the leader of the problem, who first announces the wholesale price ω of a single new type of product to the retailer with the purpose of optimizing his own expected profit. Then the retailer, as the follower, who faces the uncertain demand of the product, decides the optimal quantity Q to maximize his own expected profit. Since the retailer's order quantity is predetermined before the sales season, there are possibly two cases at the end of the sales season. If the order quantity is more than the market demand, then at the end of the selling period, the retailer sells all the leftover items at the unit salvage value s; while on the other hand, if the order quantity can't meet the market demand, the retailer will get a goodwill loss g per unit for the unmet customer demand. A revenue sharing contract (ω, α) is adopted to coordinate the system, and the percentage α of the revenue that the retailer keeps is predetermined at first. The parameters used in our model are as follows.

3.1 Notations and assumptions

We suppose that the uncertain demand faced by the retailer is denoted by a positive fuzzy variable ξ, with the support of $[0, Q]$, where Q is the ordering quantity of the product, here Q is a crisp constant number. The membership function of ξ is assumed to be continuous. Besides, we assume that the credibility density function and the credibility distribution function of ξ both exist, depicted as $\phi(x)$ and $\Phi(x)$, respectively.
The other parameters involved are given below,

c Production cost per unit.

ω Wholesale price charged by the supplier per unit.

p Selling price per unit for the sold product, which is fixed.

g Goodwill loss per unit for unmet customer demand at the end of the selling period.

s Salvage value per unit at the end of the selling cycle.

α The percentage of the total revenue that the retailer keeps.

Q Ordering quantity of the product before the selling season.

Within the framework of a revenue sharing contract, the above parameters are satisfied with the following assumptions:

(1) $\omega < c$, which implies that under the revenue sharing contract, the supplier charges the retailer a lower wholesale price in exchange for a percentage $(1 - \alpha)$ of the total revenue.

(2) $p > \omega > s$, which indicates that the retailer is rational to sell his product.

(3) The revenue sharing rate α has been settled beforehand, and generally, $\alpha \in [0, 1]$.

(4) Both the supplier and the retailer are risk-neutral, and are independent decision makers. Besides, the demand information, namely, the credibility density function and the credibility distribution function of the fuzzy variable ξ are common knowledge.

(5) $\alpha p > \omega$, which indicates that the retailer is willing to buy and sell the products when he can earn some net profit.

(6) The above cost parameters, i.e., c, p, g and s are all more than zero.

The decision variables are the supplier's wholesale price ω and the retailer's ordering quantity Q. Furthermore, assumption (5) is obtained by specific analysis as follows,
Case 1. If the market demand is equal to the retailer's order quantity, then to assure that the retailer can earn his net profit, we have $\alpha p > \omega$.

Case 2. If the market demand is less than the retailer's order quantity, for clarification, we suppose that the market demand for the product is n, and the retailer's total order quantity is N, $N > n$. We also assume that the retailer will sell the unsold products at the salvage price. To realize the retailer's profit, the following condition must be satisfied,

$$\alpha pn + \alpha s(N - n) > \omega N, \tag{26}$$

since $\alpha pn + \alpha s(N-n) = \alpha(p-s)n + \alpha sN$, thus inequality (44) can be converted into

$$\alpha(p-s)n > (\omega - \alpha s)N, \tag{27}$$

since $N > n$, it is obvious that $\alpha(p-s) > \omega - \alpha s$, consequently, we obtain $\alpha p > \omega$.

Case 3. If the market demand is more than the retailer's order quantity, we know that the retailer will sell all the products at the price p, which is similar with Case 1, thus we immediately get $\alpha p > \omega$.

In addition, the following notations are useful. $E[\Pi_r]$, $E[\Pi_s]$, $E[\Pi]$ stand for the expected profit of the retailer, the supplier and the whole system, respectively. CS refers to the centralized system, whereas DN stands for the decentralized system without contract and DC is the decentralized system with contract.

3.2 The revenue sharing model

Based on the notations and assumptions in Section 3.1, We can propose the fuzzy revenue sharing model as follows.

To achieve the coordination of the whole supply chain system, the two independent decision makers, namely, the supplier and the retailer, should act coherently so as to maximize the total profit of the whole supply chain system given by

$$E[\Pi^{DC}(Q)] = E[R_r(Q)] - Qc, \tag{28}$$

where $E[R_r(Q)]$ is the retailer's expected revenue during the selling period.

Under the revenue sharing contract, the expected value of the retailer's profit in a DN system is given by

$$\begin{aligned} E[\Pi_r^{DC}(Q)] &= \alpha\left(E[R_r(Q)]\right) - \omega Q \\ &= \alpha\left(E[u_1(\xi)] + E[u_2(\xi)]\right) - \omega Q, \end{aligned} \tag{29}$$

where $u_1(\xi)$ and $u_2(\xi)$ are the functions of the fuzzy demand ξ, with $u_1(x) = px + s(Q-x)$ and $u_2(x) = pQ - g(x-Q)$. It is obvious that $u_1(x)$ is the utility function of the retailer when the ordering quantity Q is greater than the market demand x and $u_2(x)$ is the utility function of the retailer when the ordering quantity can't meet the market demand x. It follows from assumption (ii) that $u_1(x)$ is a strictly positive increasing function of x, and $u_2(x)$ is a strictly decreasing function with respect to x. Furthermore, Thus, it follows from Theorem 2 and Theorem 3 that

$$\begin{aligned} E[\Pi_r^{DC}(Q)] &= \alpha\int_0^Q [px + s(Q-x)]\phi(x)\mathrm{d}x + \alpha\int_Q^{+\infty} [pQ - g(x-Q)]\phi(x)\mathrm{d}x - \omega Q \\ &= \alpha\int_0^Q (p-s)x\phi(x)\mathrm{d}x + \alpha\int_0^Q sQ\phi(x)\mathrm{d}x \\ &\quad +\alpha\int_Q^{+\infty} (p+g)Q\phi(x)\mathrm{d}x - \alpha\int_Q^{+\infty} gx\phi(x)\mathrm{d}x \\ &\quad -\omega Q. \end{aligned}$$

Since

$$\begin{aligned} &\int_Q^{+\infty} (p+g)Q\phi(x)\mathrm{d}x \\ &= \int_Q^0 (p+g)Q\phi(x)\mathrm{d}x + \int_0^{+\infty} (p+g)Q\phi(x)\mathrm{d}x \\ &= -\int_0^Q (p+g)Q\phi(x)\mathrm{d}x + (p+g)Q, \end{aligned}$$

and

$$\begin{aligned}
&\int_Q^{+\infty} gx\phi(x)\mathrm{d}x \\
&= \int_Q^0 gx\phi(x)\mathrm{d}x + \int_0^{+\infty} gx\phi(x)\mathrm{d}x \\
&= \int_0^{+\infty} gx\phi(x)\mathrm{d}x - \int_0^Q gx\phi(x)\mathrm{d}x,
\end{aligned}$$

so that

$$\begin{aligned}
E[\Pi_r^{DC}(Q)] &= \alpha\int_0^Q (p+g-s)x\phi(x)\mathrm{d}x \\
&\quad +\alpha\int_0^Q (s-p-g)Q\phi(x)\mathrm{d}x \\
&\quad -\alpha\int_0^{+\infty} gx\phi(x)\mathrm{d}x \\
&\quad + [\alpha(p+g)-\omega]\,Q.
\end{aligned} \tag{30}$$

Furthermore,

$$\frac{\mathrm{d}E[\Pi_r^{DC}(Q)]}{\mathrm{d}Q} = \alpha(s-p-g)\Phi(Q) + \alpha(p+g) - \omega, \tag{31}$$

and

$$\frac{\mathrm{d}^2E[\Pi_r^{DC}(Q)]}{\mathrm{d}Q^2} = \alpha(s-p-g)\phi(Q). \tag{32}$$

Proposition 1 *The retailer's expected value of his profit is concave on Q.*

Proof. On account of $\phi(Q) > 0$ and $p+g > s$, it is easily known that $\frac{\mathrm{d}^2E[\Pi_r^{DC}(Q)]}{\mathrm{d}Q^2} < 0$, thus the retailer's expected value of his profit is concave on Q.
The proposition is proved.

Proposition 2 *The retailer's optimal order quantity Q^*_{DC} and the supplier's wholesale price ω should satisfy*

$$\alpha(s-p-g)\Phi(Q^*_{DC}) + \alpha(p+g) = \omega. \tag{33}$$

Proof. Since the retailer's expected revenue is given by equation (30), so that the retailer's optimal order quantity should satisfy $\frac{\mathrm{d}E[\Pi_r^{DC}(Q)]}{\mathrm{d}Q}\ |_{Q=Q^*_{DC}}= 0$, thus it follows from equation (31) that $\alpha(s-p-g)\Phi(Q^*_{DC}) + \alpha(p+g) = \omega$.

Since the supplier is the leader, when ω is given first, it is obvious that the retailer's optimal order quantity satisfies

$$\Phi(Q^*_{DC}) = \frac{\alpha(p+g)-\omega}{\alpha(p+g-s)}. \tag{34}$$

Let Φ^{-1} denote the inverse function of Φ, then the optimal order quantity satisfies

$$Q^*_{DC} = \Phi^{-1}\left(\frac{\alpha(p+g)-\omega}{\alpha(p+g-s)}\right).$$

The relationship between the parameters, is shown by equation (33), and when the supplier has already decided the whole sale price, the retailer's optimal order quantity should satisfy equation (34).

Furthermore, we have the following discussion on the relationship between the contract parameters, ω and α. To achieve channel coordination, it is necessary that the optimal quantity Q^*_{DC}

chosen by the retailer should corresponds to the order quantity Q^*_{CS} that maximizes the total profit of the whole supply chain, namely,

$$E[\Pi^{CS}(Q)] = E[R_r(Q)] - Qc. \tag{35}$$

It follows from equation (29) that the expected value of the retailer's profit in a DN system is

$$E[\Pi^{DC}_r(Q)] = \alpha\left(E[R_r(Q)]\right) - \omega Q, \tag{36}$$

thus the optimal quantity Q^*_{DC} chosen by the retailer must satisfy $\frac{\mathrm{d}E[\Pi^{DC}_r(Q)]}{\mathrm{d}Q}\mid_{Q=Q^*_{DC}} = 0$, namely,

$$\alpha\frac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q}\mid_{Q=Q^*_{DC}} = \omega. \tag{37}$$

What's more, the order quantity Q^*_{CS} satisfies

$$\frac{\mathrm{d}E[\Pi^{CS}(Q)]}{\mathrm{d}Q}\mid_{Q=Q^*_{CS}} = 0, \tag{38}$$

i.e.,

$$\frac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q}\mid_{Q=Q^*_{CS}} = c. \tag{39}$$

Since $Q^*_{DC} = Q^*_{CS}$, thus by matching equation (37) and equation (39), we know that under the revenue sharing contract, the supplier's optimal wholesale unit price is

$$\omega = \alpha c. \tag{40}$$

The above equation shows that although salvage value and goodwill loss are considered, the contract parameters ω and α are still in the relationship of $\omega = \alpha c$, as previously discussed in Cachon and Lariviere's work. However, considering that the former researchers haven't made further analysis why the optimal revenue sharing contract is in the form of $\omega = \alpha c$, with which the supply chain's total profit as well as the individual players' profit is maximized, we will present the analysis as follows.

We first suppose that $\omega > \alpha c$, if this happens, it follows from equations (37) and (39) that

$$\alpha\frac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q}\mid_{Q=Q^*_{CS}} < \alpha\frac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q}\mid_{Q=Q^*_{DC}},$$

namely,

$$\frac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q}\mid_{Q=Q^*_{CS}} < \frac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q}\mid_{Q=Q^*_{DC}},$$

since

$$\frac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q} = (s-p-g)\Phi(Q) + (p+g),$$

and

$$\frac{\mathrm{d}^2E[R_r(Q)]}{\mathrm{d}Q^2} = (s-p-g)\phi(Q) < 0,$$

which indicates that $\frac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q}$ is a decreasing function of Q, immediately, we know $Q^*_{CS} > Q^*_{DC}$. Since the total profit of the supply chain, regardless of the relationship between the players, can be calculated by the quantity of the products sold to the customers, thus the more products sold in the whole supply chain, the more profit the supply chain gains. Obviously, when $Q^*_{CS} > Q^*_{DC}$, the decentralized supply chain sells less products than the centralized supply chain , the whole supply chain doesn't get the maximum profit.

On the contrary, if $\omega < \alpha c$, we immediately get $Q^*_{CS} < Q^*_{DC}$. We might think that due to the same reason that the more the supply chain sells, the more profit the supply chain gets, this contract may be better than $\omega = \alpha c$. However, we have to notice that the contract proposed should not only maximize the supply chain's total profit, but also should uphold the individual players' own profit. So in the case when $\omega > \alpha c$, we have the following discussions. For the retailer, the profit gained when $Q = Q^*_{DC}$ is $E[\Pi_r^{DC}(Q)] \mid_{Q=Q^*_{DC}}$. By Proposition 1, we know that $E[\Pi_r^{DC}(Q)]$ is concave on Q, which means that $\dfrac{dE[\Pi_r^{DC}(Q)}{dQ}$ is a decreasing function. Since $\dfrac{dE[\Pi_r^{DC}(Q)}{dQ} \mid_{Q=Q^*_{DC}} = 0$, and $Q^*_{CS} < Q^*_{DC}$, it implies that when $Q \in [0, Q^*_{DC}]$, the retailer's profit function is increasing. Similarly, when the order quantity is with in the set of $[Q^*_{DC}, +\infty)$, the retailer's profit is decreasing. So the retailer gets his optimal profit when his optimal order quantity is $Q = Q^*_{DC}$.

However, for the supplier, his profit is $E[\Pi_s^{DC}] = (1-\alpha)\left(E[R_r(Q)]\right) + \omega Q - cQ$. Since

$$\frac{\mathrm{d}E[\Pi_s^{DC}]}{\mathrm{d}Q} = (1-\alpha)\frac{\mathrm{d}E[R_r^Q]}{\mathrm{d}Q} + \omega - c,$$

and

$$\frac{\mathrm{d}^2E[\Pi_s^{DC}]}{\mathrm{d}Q^2} = (1-\alpha)\frac{\mathrm{d}^2E[R_r^Q]}{\mathrm{d}Q^2} < 0,$$

we know that $\dfrac{\mathrm{d}E[\Pi_s^{DC}]}{\mathrm{d}Q}$ is a decreasing function. Assume the supplier's expected order quantity is Q^*, which satisfies $\dfrac{\mathrm{d}E[\Pi_s^{DC}]}{\mathrm{d}Q} = 0$, then we get $\dfrac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q} \mid_{Q=Q^*} = \dfrac{c-w}{1-\alpha}$. On account of the retailer's optimal order quantity Q^*_{DC} satisfies $\dfrac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q} \mid_{Q=Q^*_{DC}} = \dfrac{\omega}{\alpha}$, we have

$$\frac{\omega}{\alpha} - \frac{c-\omega}{1-\alpha} = \frac{\omega - c\alpha}{\alpha(1-\alpha)} < 0,$$

which implies that

$$\frac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q} \mid_{Q=Q^*_{DC}} < \frac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q} \mid_{Q=Q^*},$$

since $\dfrac{\mathrm{d}E[R_r(Q)]}{\mathrm{d}Q}$ is a decreasing function, we get $Q^* < Q^*_{DC}$, which indicates that the supplier's profit is maximized when $Q = Q^*$. What's more, when $Q > Q^*$, the supplier's profit is decreasing, while when $Q < Q^*$, the supplier's profit is increasing. To sum up, although both the whole supply chain's profit and the retailer's profit are maximized, the supplier's profit couldn't be maximized.

The above analysis shows that in order to achieve the coordination of the whole supply chain, as well as ensuring that each player get his optimal profit, the revenue sharing adopted should be in the form of $\omega = \alpha c$. Furthermore, we might look at it from another point of view. The whole sale price the supplier charged with the retailer is some portion of the total production cost, which can be denoted by $\omega = \lambda c$, here λ can be interpreted as the transfer cost rate from the supplier to the retailer, from $\omega = \alpha c$, we immediately know that $\lambda = \alpha$, which means when the retailer bears the percentage of λ of the total cost, he in turn gets α percentage of the total profit. This is reasonable because for the rational player, the profit should be in proportion to his efforts.

The above analysis indicates that under the revenue sharing contract, the supplier will offer the retailer a wholesale price lower than his unit cost ($\omega = \alpha c$), however, in return, he receives a fixed quota $(1-\alpha)$ of the retailer's selling revenue, namely,

$$E[\Pi_s^{DC}] = (1-\alpha)\left(E[R_r(Q)]\right) + \omega Q - cQ,$$

since

$$\begin{aligned} E[R_r(Q)] &= \int_0^Q (p+g-s)x\phi(x)\mathrm{d}x \\ &+ \int_0^Q (s-p-g)Q\phi(x)\mathrm{d}x \\ &- \int_0^{+\infty} gx\phi(x)\mathrm{d}x \\ &+ \left[(p+g)\right]Q. \end{aligned}$$

thus

$$
\begin{aligned}
E[\Pi_s^{DC}] &= (1-\alpha)\int_0^Q (p+g-s)x\phi(x)\mathrm{d}x \\
&+(1-\alpha)\int_0^Q (s-p-g)Q\phi(x)\mathrm{d}x \\
&-(1-\alpha)\int_0^{+\infty} gx\phi(x)\mathrm{d}x \\
&+\left[(1-\alpha)(p+g)+\omega-c\right]Q.
\end{aligned}
$$

Furthermore,

$$\frac{\mathrm{d}E[\Pi_s^{DC}]}{\mathrm{d}Q} = (1-\alpha)(s-p-g)\Phi(Q) + [(1-\alpha)(p+g)+\omega-c], \tag{41}$$

and

$$\frac{\mathrm{d}^2E[\Pi_s^{DC}]}{\mathrm{d}Q^2} = (1-\alpha)(s-p-g)\phi(Q). \tag{42}$$

Since Q is determined by the retailer, and under the revenue sharing contract that $\omega = \alpha c$, the retailer's profit reaches optima, and the whole supply chain gets a maximum overall profit, if the supplier's profit can also be maximized, we can say that the whole supply chain is coordinated under this contract.

To be more specific, we present the following proposition.

Proposition 3 *The supplier's expected value of his profit is maximized when the optimal order quantity of the supplier is Q^*_{DC}.*

Proof. Assume that the optimal quantity the supplier expects from the retailer is $Q^{*'}_{DC}$, to get the optimal profit, $Q^{*'}_{DC}$ must satisfy $\frac{\mathrm{d}E[\Pi_s^{DC}]}{\mathrm{d}Q}\mid_{Q=Q^{*'}_{DC}} = 0$, namely,

$$(1-\alpha)(s-p-g)\Phi(Q^{*'}_{DC}) = c-\omega-(1-\alpha), \tag{43}$$

thus, given ω, it is evident that the optimal order quantity the supplier expects should satisfy

$$\Phi(Q^{*'}_{DC}) = \frac{(p+g)(1-\alpha)+\omega-c}{(1-\alpha)(p+g-s)}. \tag{44}$$

By substituting $\omega = \alpha c$ into the above equation, we obtain

$$\Phi(Q^{*'}_{DC}) = \frac{\alpha(p+g)-\omega}{\alpha(p+g-s)}. \tag{45}$$

Comparing equation (45) with equation (34), we immediately conclude that the optimal order quantity the retailer chooses is equal to the supplier's expected order quantity, i.e., $Q^{*'}_{DC} = Q^*_{DC}$, which indicates that the retailer's expected profit and the supplier's expected profit can reach their optimum on Q^*_{DC}. The proposition is proved.

Ultimately, the total profit of the decentralized supply chain under the revenue sharing contract is

$$
\begin{aligned}
E[\Pi^{DC}(Q)] = E[\Pi^{CS}(Q)] &= \int_0^Q (p+g-s)x\phi(x)\mathrm{d}x \\
&+\int_0^Q (s-p-g)Q\phi(x)\mathrm{d}x \\
&-\int_0^{+\infty} gx\phi(x)\mathrm{d}x \\
&+\left[(p+g)-c\right]Q.
\end{aligned}
$$

4 Conclusions

This paper investigated the coordination of a single-period decentralized supply chain, formed by a risk-neutral supplier and a risk-neutral retailer, where the uncertain market demand of the selling product was denoted by a fuzzy variable. With a revenue sharing contract considering salvage value and goodwill loss, a fuzzy supply chain model was formulated, which aimed at maximizing the supply chain's total profit and allocating the coordination profit among the supplier and the retailer. By analyzing the proposed model, the relationship between the revenue contract parameters were given, and the reasons why the optimal contract parameters should satisfy $\omega = \alpha c$ were demonstrated, the cases when the condition was not satisfied were analyzed to further confirm the rationality of the optimal contract.

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